

Generalized Analysis of Convergence of Absolute Trust in Peer to Peer Networks

Sateesh Kumar Awasthi and Yatindra Nath Singh, *Senior Member, IEEE*

Abstract—Open and anonymous nature of peer to peer networks provides an opportunity to malicious peers to behave unpredictably in the network. This leads the lack of trust among the peers. To control the behavior of peers in the network, reputation system can be used. In a reputation system, aggregation of trust is a primary issue. Algorithm for aggregation of trust should be designed such that, it can converge to a certain finite value. Absolute Trust is one of the algorithm, which is used for the aggregation of trust in peer to peer networks. In this letter, we present the generalized analysis of convergence of the Absolute Trust algorithm.

Index Terms—Non-negative matrix, Eigenvector, Matrix Norm.

I. INTRODUCTION

REPUTATION systems have been proposed by many authors in the recent past [1], [5], [6], [7], [8], to prevent the attacks by rogue peers. Absolute Trust [1] is one such model. This model can characterize the past behavior of peers in the network. It can be implemented as a truly distributed system. In this model, peers evaluate each other locally, and the local trust for each other is aggregated in the whole network. The aggregated trust is called global trust. The global trust is evaluated recursively.

In recursive solution of any equation, error in each iteration must reduce and for large number of iteration it should tend to zero. This will guarantee the uniqueness of the solution. It was shown in [1] that if error in global trust is less compare to the actual solution, then it will converge to zero. But analysis for large error was not presented. In this letter, we will show that in any step, if error is very large compared to the actual solution then it will converge much faster in that step.

II. PEER TO PEER MODEL AND ABSOLUTE TRUST

Let there be N peers in a peer to peer network. In this network, peer i can be evaluated by peer j based on service provided by peer i in the past. Evaluated value T_{ji} can be represented by a number from one to ten. One is for worst service and ten is for best service. If there is no interaction between peers, T_{ji} will be zero. T_{ji} is called local trust of peer i evaluated by peer j . All local trust values evaluated by various nodes can be aggregated in the whole network. Aggregated global trust of peer i can be given by [1]

$$t_i = \left[\left(\frac{\sum_{j \in S_i} T_{ji} t_j}{\sum_{j \in S_i} t_j} \right)^p \cdot \left(\frac{\sum_{j \in S_i} t_j^2}{\sum_{j \in S_i} t_j} \right)^q \right]^{\frac{1}{p+q}}. \quad (1)$$

Here, S_i is the set of peers evaluating the service of peer i , t_j is global trust of peer j , and p, q are suitably chosen constants. Equation 1 can be rearranged as follows (see [1]).

$$\begin{aligned} t_i &= \left[\left(\frac{\mathbf{e}_i \mathbf{T}^t \mathbf{t}}{\mathbf{e}_i \mathbf{C} \mathbf{t}} \right)^p \left(\frac{\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\mathbf{t}) \cdot \mathbf{t}}{\mathbf{e}_i \mathbf{C} \mathbf{t}} \right)^q \right]^{\frac{1}{p+q}} \\ &= \left[\left(\frac{\mathbf{e}_i \mathbf{T}^t \mathbf{t}}{\mathbf{e}_i \mathbf{C} \mathbf{t}} \right)^{\frac{1}{1+\alpha}} \left(\frac{\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\mathbf{t}) \cdot \mathbf{t}}{\mathbf{e}_i \mathbf{C} \mathbf{t}} \right)^{\frac{\alpha}{1+\alpha}} \right] \end{aligned}$$

Here, \mathbf{t} is global trust vector. Its i^{th} entry is a global trust of peer i . $\text{diag}(\mathbf{t})$ is a diagonal matrix with its ii entry as t_i and other entry as zero. \mathbf{T} is trust matrix with its element T_{ij} as a local trust of peer j evaluated by peer i . \mathbf{T}^t is transpose of matrix \mathbf{T} . \mathbf{C} is incidence matrix corresponding to matrix \mathbf{T}^t i.e. $C_{ij} = 1$ if $T_{ji} > 0$ otherwise $C_{ij} = 0$. \mathbf{e}_i is the row vector with its i^{th} entry as '1' and all the other entry as zero. Here, $\alpha = q/p$.

III. ANALYSIS OF CONVERGENCE OF ABSOLUTE TRUST

A. Center Point of the Matrix

Definition 1. Center point of non-negative matrix \mathbf{T}^t can be defined as the column vector \mathbf{t} . Where, its i^{th} element t_i will be $(\mathbf{e}_i \mathbf{T}^t \mathbf{t} / \mathbf{e}_i \mathbf{C} \mathbf{t})$.

Definition 2. A non-negative matrix \mathbf{M} is said to be mutually exclusive with a non-negative matrix \mathbf{N} , if $M_{ij} > 0$ implies $N_{ij} = 0$. It also implies that when $N_{ij} > 0$ then $M_{ij} = 0$

Lemma 1. Center point of any non-negative, irreducible matrix \mathbf{T}^t is unique and can be calculated by an iterative function

$$\mathbf{t}^k = \phi_1(\mathbf{t}^{k-1}) = [\text{diag}(d_1, d_2, \dots, d_N)]^{-1} \cdot \mathbf{T}^t \cdot \mathbf{t}^{k-1}$$

where $d_i = \mathbf{e}_i \mathbf{C} \mathbf{t}$

Proof. i^{th} element of iterative function $\phi_1(\mathbf{t}^{k-1})$ is

$$t_i^k = \frac{(\mathbf{e}_i \mathbf{T}^t \cdot \mathbf{t}^{k-1})}{(\mathbf{e}_i \mathbf{C} \cdot \mathbf{t}^{k-1})} \quad (2)$$

Let t_i^k and t_i^{k-1} are, far from actual solution t_i by δt_i^k and δt_i^{k-1} respectively, then

$$\begin{aligned} t_i + \delta t_i^k &= \left[\frac{(\mathbf{e}_i \mathbf{T}^t \cdot (\mathbf{t} + \delta \mathbf{t}^{k-1}))}{(\mathbf{e}_i \mathbf{C} \cdot (\mathbf{t} + \delta \mathbf{t}^{k-1}))} \right] \\ &= \left[\frac{(\mathbf{e}_i \mathbf{T}^t \cdot \mathbf{t})}{(\mathbf{e}_i \mathbf{C} \cdot \mathbf{t})} \right] \left[\frac{(1 + \frac{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \mathbf{t}})}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{C} \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}})} \right]. \end{aligned}$$

Sateesh Kumar Awasthi is with the Department of Electrical Engineering, Indian Institute of Technology, Kanpur, India, e-mail: (sateesh@iitk.ac.in)

Yatindra Nath Singh is with the Department of Electrical Engineering, Indian Institute of Technology, Kanpur, India, e-mail: (ynsingh@iitk.ac.in)

From definition 1, $(\mathbf{e}_i \mathbf{T}^t \mathbf{t} / \mathbf{e}_i \mathbf{C} \mathbf{t})$ is i^{th} element of center point of matrix \mathbf{T}^t . So we can write,

$$\begin{aligned} t_i + \delta t_i^k &= t_i \cdot \left[\frac{(1 + \frac{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \mathbf{t}})}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{C} \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}})} \right] \\ \delta t_i^k &= t_i \cdot \left[\frac{(1 + \frac{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \mathbf{t}})}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{C} \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}})} - 1 \right] \\ \delta t_i^k &= \frac{t_i}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{C} \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}})} \cdot \left[\frac{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \mathbf{t}} - \frac{\mathbf{e}_i \cdot \mathbf{C} \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}} \right] \\ &= \frac{1}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{C} \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}})} \cdot \left[\frac{t_i \mathbf{e}_i \cdot \mathbf{T}^t}{\mathbf{e}_i \cdot \mathbf{T}^t \cdot \mathbf{t}} - \frac{t_i \mathbf{e}_i \cdot \mathbf{C}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}} \right] \cdot \delta \mathbf{t}^{k-1} \\ &= \frac{1}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{C} \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}})} \cdot [\mathbf{A}_i - \mathbf{B}_i] \cdot \delta \mathbf{t}^{k-1} \\ &= f_i(\delta \mathbf{t}^{k-1}) \cdot [\mathbf{A}_i - \mathbf{B}_i] \cdot \delta \mathbf{t}^{k-1} \end{aligned}$$

where \mathbf{A}_i and \mathbf{B}_i are i^{th} row of $N \times N$ matrices \mathbf{A} and \mathbf{B} respectively. It can be observed easily that

$$\mathbf{A} \cdot \mathbf{t} = \mathbf{t}$$

and

$$\mathbf{B} \cdot \mathbf{t} = \mathbf{t}.$$

Matrices \mathbf{A}, \mathbf{B} have non zero elements at same position as matrix \mathbf{T}^t , hence \mathbf{A}, \mathbf{B} are also irreducible. Therefore spectral radius of \mathbf{A}, \mathbf{B} will be '1'(see[2]).

Now

$$f_i(\delta \mathbf{t}^{k-1}) = \frac{1}{(1 + \frac{\mathbf{e}_i \cdot \mathbf{C} \cdot \delta \mathbf{t}^{k-1}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}})}$$

If $\delta \mathbf{t}^{k-1} \ll \mathbf{t}$ then $f_i(\delta \mathbf{t}^{k-1}) \approx 1$

Hence,

$$\delta \mathbf{t}^k = [\mathbf{A} - \mathbf{B}] \cdot \delta \mathbf{t}^{k-1}$$

$$\lim_{k \rightarrow \infty} \delta \mathbf{t}^k = \lim_{k \rightarrow \infty} [\mathbf{A} - \mathbf{B}]^k \delta \mathbf{t}^0 = \mathbf{0}$$

(see Theorem 1 in [1]) here $\delta \mathbf{t}^0$ is initial error in \mathbf{t}

Now, for the case when $\delta \mathbf{t}^{k-1} > \mathbf{t}$ then $f_i(\delta \mathbf{t}^{k-1}) < 1$, in each step, δt_i^k will decrease more rapidly. If $\delta \mathbf{t}^{k-1} \approx \mathbf{t}$ then $f_i(\delta \mathbf{t}^{k-1}) \approx 1/2$, in this case each δt_i^k will reduced to half in k^{th} step.

Hence we can conclude that center point of any non negative, irreducible matrix can be calculated by above iterative function. Error in each step will depend upon the error in past step. Error will reduce very fast, if it is far from actual solution in an step and after large iterations it will go to zero. \square

Lemma 2. If vector \mathbf{t} is the center point of matrix $\mathbf{C} \cdot \text{diag}(\mathbf{t})$, then center point will lie on the vector \mathbf{e} . It can be calculated by iterative function

$$\mathbf{t}^k = \phi_2(\mathbf{t}^{k-1}) = [\text{diag}(d_1, d_2, \dots, d_N)]^{-1} \cdot \mathbf{C} \cdot \text{diag}(\mathbf{t}^{k-1}) \cdot \mathbf{t}^{k-1}$$

where \mathbf{e} is a vector with each element as '1' and $d_i = \mathbf{e}_i \mathbf{C} \mathbf{t}$.

Proof. Iterative function can be written as

$$\begin{aligned} \mathbf{t}^k &= \mathbf{M}(\mathbf{k} - 1) \cdot \mathbf{t}^{k-1} \\ &= \mathbf{M}(\mathbf{k} - 1) \cdot \mathbf{M}(\mathbf{k} - 2) \dots \mathbf{M}(0) \cdot \mathbf{t}^0 \end{aligned}$$

where i^{th} row of matrix $\mathbf{M}(\mathbf{k})$ is $(\frac{\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\mathbf{t}^k)}{\mathbf{e}_i \mathbf{C} \cdot \mathbf{t}^k})$. We can easily prove that for matrix $\mathbf{M}(\mathbf{k})$, sum of its each row is one. Hence it has '1' as an eigen value. The corresponding eigen vector will be \mathbf{e} . For positive initial guess of \mathbf{t}^0 , all matrices $\mathbf{M}(\mathbf{k})$ will be non-negative and irreducible. So we can conclude that spectral radius of all $\mathbf{M}(\mathbf{k})$ is '1'. Hence (see Lemma 2 in [1])

$$\lim_{k \rightarrow \infty} \mathbf{t}^k = \lim_{k \rightarrow \infty} \mathbf{M}(\mathbf{k} - 1) \cdot \mathbf{M}(\mathbf{k} - 2) \dots \mathbf{M}(0) \cdot \mathbf{t}^0 = \mathbf{e}$$

$$\lim_{k \rightarrow \infty} \mathbf{t}^k = \lim_{k \rightarrow \infty} \phi_2(\mathbf{t}^{k-1}) = \mathbf{e}$$

\square

B. Properties of Center Point

Property 1. If center point of matrix \mathbf{M} is \mathbf{t} then center point of matrix $k\mathbf{M}$ will be $k\mathbf{t}$. Where k is any arbitrary scalar.

Proof. Let t_i is i^{th} element of \mathbf{t} then

$$t_i = \frac{\mathbf{e}_i \cdot \mathbf{M} \cdot \mathbf{t}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}}$$

or

$$kt_i = \frac{\mathbf{e}_i \cdot k\mathbf{M} \cdot \mathbf{t}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}}$$

or

$$(kt_i) = \frac{\mathbf{e}_i \cdot (k\mathbf{M}) \cdot (kt)}{\mathbf{e}_i \cdot \mathbf{C} \cdot (kt)}$$

hence $k\mathbf{t}$ is center point of matrix $k\mathbf{M}$ \square

Property 2. If all entries of the $N \times N$ matrix \mathbf{M} are positive ($M_{ij} > 0 \forall i, j$) then center point will lie on the principle eigen vector of matrix.

Proof. If all the entry of matrix \mathbf{M} are positive then

$$\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t} = \sum_{j=1}^N t_j = \lambda \quad \forall i$$

Here λ is a constant. Further,

$$t_i = \frac{\mathbf{e}_i \cdot \mathbf{M} \cdot \mathbf{t}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}} = \frac{\mathbf{e}_i \cdot \mathbf{M} \cdot \mathbf{t}}{\lambda}.$$

Hence $\mathbf{M} \mathbf{t} = \lambda \mathbf{t}$. Hence λ will be spectral radius and \mathbf{t} will be principle eigen vector of matrix \mathbf{M} (see[2]) \square

Property 3. If center point of m non-negative, irreducible and mutually exclusive matrices $\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_m$ are same, then the center point of their sum will also be the same.

Proof. Let the incidence matrix corresponding to matrices $\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_m$ are $\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_m$ respectively. Now if $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \dots + \mathbf{M}_m$ then incidence matrix corresponding to matrix \mathbf{M} will be $\mathbf{C} = \mathbf{C}_1 + \mathbf{C}_2 + \dots + \mathbf{C}_m$ because matrices $\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_m$ are mutually exclusive. Let i^{th} element of center point of matrices $\mathbf{M}_1, \mathbf{M}_2, \dots, \mathbf{M}_m$ be t_i then

$$t_i = \frac{\mathbf{e}_i \cdot \mathbf{M}_j \cdot \mathbf{t}}{\mathbf{e}_i \cdot \mathbf{C}_j \cdot \mathbf{t}} \quad \forall j$$

or

$$t_i \mathbf{e}_i \cdot \mathbf{C}_j \cdot \mathbf{t} = \mathbf{e}_i \cdot \mathbf{M}_j \cdot \mathbf{t}$$

taking summation on both side w.r.t. j

$$\begin{aligned} \sum_{j=1}^m t_i \mathbf{e}_i \cdot \mathbf{C}_j \cdot \mathbf{t} &= \sum_{j=1}^m \mathbf{e}_i \cdot \mathbf{M}_j \cdot \mathbf{t} \\ t_i \mathbf{e}_i \cdot \left(\sum_{j=1}^m \mathbf{C}_j \right) \cdot \mathbf{t} &= \mathbf{e}_i \cdot \left(\sum_{j=1}^m \mathbf{M}_j \right) \cdot \mathbf{t} \\ t_i \mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t} &= \mathbf{e}_i \cdot \mathbf{M} \cdot \mathbf{t} \end{aligned}$$

or

$$t_i = \frac{\mathbf{e}_i \cdot \mathbf{M} \cdot \mathbf{t}}{\mathbf{e}_i \cdot \mathbf{C} \cdot \mathbf{t}}$$

hence \mathbf{t} is also the center point of matrix \mathbf{M} .

□

C. Convergence of Absolute Trust for Large Error in Initial Guess

It was stated in [1] that global trust in equation 1 can be calculated by iterative function

$$\mathbf{t}^k = \phi(\mathbf{t}^{k-1}) = [\text{diag}(d_1, d_2, \dots, d_N) \mathbf{T}^t \cdot \mathbf{t}^{k-1}]^{\frac{1}{1+\alpha}}$$

where $d_i = \left[\frac{(\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\mathbf{t}^{k-1}) \cdot \mathbf{t}^{k-1})^\alpha}{(\mathbf{e}_i \mathbf{C} \mathbf{t}^{k-1})^{(1+\alpha)}} \right]$. Proof was derived only for the small error that is, if initial guess of \mathbf{t} is very close to the actual solution. However global trust can be calculated for any positive initial guess. In this subsection we will show that it will converge faster in any step if error is large compared to \mathbf{t} .

Let t_i^k and t_i^{k-1} are, far from actual solution t_i by δt_i^k and δt_i^{k-1} respectively, then

$$\begin{aligned} t_i + \delta t_i^k &= \left[\frac{(\mathbf{e}_i \mathbf{T}^t \cdot (\mathbf{t} + \delta \mathbf{t}^{k-1}))}{(\mathbf{e}_i \mathbf{C} (\mathbf{t} + \delta \mathbf{t}^{k-1}))} \right]^{\frac{1}{1+\alpha}} \\ &\quad \left[\frac{(\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\mathbf{t} + \delta \mathbf{t}^{k-1}) \cdot (\mathbf{t} + \delta \mathbf{t}^{k-1}))}{(\mathbf{e}_i \mathbf{C} (\mathbf{t} + \delta \mathbf{t}^{k-1}))} \right]^{\frac{\alpha}{1+\alpha}} \end{aligned}$$

If error $\delta \mathbf{t}^{k-1} > \mathbf{t}$ then we can approximate $\mathbf{t} + \delta \mathbf{t}^{k-1} \approx \delta \mathbf{t}^{k-1}$ hence

$$\delta t_i^k = \left[\frac{(\mathbf{e}_i \mathbf{T}^t \cdot \delta \mathbf{t}^{k-1})}{(\mathbf{e}_i \mathbf{C} \cdot \delta \mathbf{t}^{k-1})} \right]^{\frac{1}{1+\alpha}} \cdot \left[\frac{(\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\delta \mathbf{t}^{k-1}) \cdot \delta \mathbf{t}^{k-1})}{(\mathbf{e}_i \mathbf{C} \cdot \delta \mathbf{t}^{k-1})} \right]^{\frac{\alpha}{1+\alpha}}$$

Using Young's Inequality [4], i.e.

$$c \cdot d \leq \frac{1}{1+\alpha} c^{1+\alpha} + \frac{\alpha}{1+\alpha} d^{(1+\alpha)/\alpha};$$

and taking

$$c = \left[\frac{(\mathbf{e}_i \mathbf{T}^t \cdot \delta \mathbf{t}^{k-1})}{(\mathbf{e}_i \mathbf{C} \cdot \delta \mathbf{t}^{k-1})} \right]^{\frac{1}{1+\alpha}}$$

and

$$d = \left[\frac{(\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\delta \mathbf{t}^{k-1}) \cdot \delta \mathbf{t}^{k-1})}{(\mathbf{e}_i \mathbf{C} \cdot \delta \mathbf{t}^{k-1})} \right]^{\frac{\alpha}{1+\alpha}},$$

We can write

$$\begin{aligned} \delta t_i^k &\leq \frac{1}{1+\alpha} \left[\frac{(\mathbf{e}_i \mathbf{T}^t \cdot \delta \mathbf{t}^{k-1})}{(\mathbf{e}_i \mathbf{C} \cdot \delta \mathbf{t}^{k-1})} \right] + \\ &\quad \frac{\alpha}{1+\alpha} \left[\frac{(\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\delta \mathbf{t}^{k-1}) \cdot \delta \mathbf{t}^{k-1})}{(\mathbf{e}_i \mathbf{C} \cdot \delta \mathbf{t}^{k-1})} \right]. \end{aligned}$$

$$\delta \mathbf{t}^k \leq \frac{1}{1+\alpha} \phi_1(\delta \mathbf{t}^{k-1}) + \frac{\alpha}{1+\alpha} \phi_2(\delta \mathbf{t}^{k-1}).$$

Here $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are as defined in Lemma 1 and in Lemma 2 respectively.

$$\delta \mathbf{t}^k \leq \frac{1}{1+\alpha} \mathbf{M}'_1 \cdot \delta \mathbf{t}^{k-1} + \frac{\alpha}{1+\alpha} \mathbf{M}'_2 \cdot \delta \mathbf{t}^{k-1} \quad (3)$$

It is convex combination of iterative function ϕ_1 and ϕ_2 . i^{th} row of matrix \mathbf{M}'_2 is $\left[\frac{(\mathbf{e}_i \mathbf{C} \cdot \text{diag}(\delta \mathbf{t}^{k-1}))}{(\mathbf{e}_i \mathbf{C} \cdot \delta \mathbf{t}^{k-1})} \right]$ and sum of each row is one. Hence ∞ -norm of matrix \mathbf{M}'_2 is $|\mathbf{M}'_2|_\infty = 1$ using the property of norm [3]

$$|\mathbf{M}'_2 \cdot \delta \mathbf{t}^{k-1}|_\infty \leq |\mathbf{M}'_2|_\infty \cdot |\delta \mathbf{t}^{k-1}|_\infty = |\delta \mathbf{t}^{k-1}|_\infty \quad (4)$$

Function ϕ_1 is converging function toward center point. It is shown in Lemma 1 that in any step, if \mathbf{t}^k is very far from the center point then it will tend toward the center point very rapidly. Hence for $\delta \mathbf{t}^{k-1} > \mathbf{t}$

$$\phi_1(\delta \mathbf{t}^{k-1}) < \delta \mathbf{t}^{k-1}$$

or

$$|\mathbf{M}'_1 \cdot \delta \mathbf{t}^{k-1}|_\infty < |\delta \mathbf{t}^{k-1}|_\infty \quad (5)$$

Now taking ∞ -norm on both side of equation 3

$$\begin{aligned} |\delta \mathbf{t}^k|_\infty &\leq \left| \frac{1}{1+\alpha} \mathbf{M}'_1 \cdot \delta \mathbf{t}^{k-1} + \frac{\alpha}{1+\alpha} \mathbf{M}'_2 \cdot \delta \mathbf{t}^{k-1} \right|_\infty \\ &\leq \frac{1}{1+\alpha} |\mathbf{M}'_1 \cdot \delta \mathbf{t}^{k-1}|_\infty + \frac{\alpha}{1+\alpha} |\mathbf{M}'_2 \cdot \delta \mathbf{t}^{k-1}|_\infty \end{aligned}$$

Using equation 4 and 5,

$$|\delta \mathbf{t}^k|_\infty < \frac{1}{1+\alpha} |\delta \mathbf{t}^{k-1}|_\infty + \frac{\alpha}{1+\alpha} |\delta \mathbf{t}^{k-1}|_\infty = |\delta \mathbf{t}^{k-1}|_\infty$$

Hence

$$|\delta \mathbf{t}^k|_\infty < |\delta \mathbf{t}^{k-1}|_\infty$$

Therefore error in every step will decrease. In any step, it will decrease faster if \mathbf{t}^k is very far from actual solution. Speed of convergence depends upon the α . For lower α , impact of ϕ_2 will be lower and ϕ_1 will dominate the speed of convergence. Hence for smaller α speed of convergence will be high.

IV. NUMERICAL RESULTS

In order to verify what has been discussed in earlier sections, we have taken the values of the trust matrix \mathbf{T} as

$$\begin{bmatrix} 0 & 5 & 6 & 6 \\ 8 & 0 & 5 & 5 \\ 5 & 6 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix}.$$

The value of center point and the global trust in each iteration is calculated and shown in Table I, II, III, IV and V.

Table I: Center point in each iteration, when initial guess is close to center point

i	1	2	3	4
t^0	1.0000	2.0000	3.0000	4.0000
t^1	6.2000	4.8750	5.3333	3.6667
t^2	6.4327	5.1096	5.5598	4.4027
t^3	6.4367	5.0706	5.5573	4.4008
t^4	6.4313	5.0705	5.5594	4.4002
t^5	6.4310	5.0707	5.5592	4.3994
t^6	6.4311	5.0708	5.5591	4.3994
t^7	6.4311	5.0708	5.5591	4.3994

Table II: Center point in each iteration, when initial guess is very far from center point

i	1	2	3	4
t^0	100.0000	300.0000	200.0000	100.0000
t^1	6.8000	5.2500	5.2500	4.1667
t^2	6.5000	5.0668	5.5643	4.4827
t^3	6.4298	5.0654	5.5620	4.4050
t^4	6.4299	5.0706	5.5593	4.3987
t^5	6.4310	5.0708	5.5591	4.3993
t^6	6.4311	5.0708	5.5591	4.3994
t^7	6.4311	5.0708	5.5591	4.3994

A. Convergence of the Center Point

The convergence of center point of matrix T^t is shown in Table I and II. In Table I initial guess $t^0 = [1 \ 2 \ 3 \ 4]^t$, it is close to the center point and in Table II initial guess $t^0 = [100 \ 300 \ 200 \ 100]^t$, which is significantly far from center point. But we can see in both the cases that it will converge in seven iterations. In the latter case, error is very large in 0^{th} step and it becomes less than t with in one step.

B. Convergence of the Global Trust

Impact of initial guess and parameter α on the convergence of global trust is shown in Table III, IV and V. In Table III and IV α is taken as $1/3$ but initial guess is different. Again we can see that for large initial guess of global trust it takes only one more iteration to converge to final value. It converge very fast when error(δt) is very large compare to global trust(t)

In Table III and V initial guess is taken same but α is different and we can see that for $\alpha = 1/6$ it converges only in eight iterations.

V. CONCLUSION

In this letter, we analyzed the convergence of global trust given by equation 1. We have shown that in recursive calculation of global trust, error will decrease even if initial guess is very far from the actual solution. In any step, convergence is faster if error is larger. We have shown that speed of convergence depends upon the value of α . For smaller α it will converge more faster.

REFERENCES

- [1] S. K. Awasthi and Y. N. Singh, "Absolute Trust: Algorithm for Aggregation of Trust in Peer to Peer Network", <http://arxiv.org/abs/1601.01419>

Table III: Global trust in each iteration, when initial guess is close to global trust ($\alpha = 1/3$)

i	1	2	3	4
t^0	1.0000	2.0000	3.0000	4.0000
t^1	4.9893	4.4051	3.9876	3.2749
t^2	5.8801	4.8299	5.3148	4.4838
t^3	6.0621	5.1110	5.5131	4.6039
t^4	6.1418	5.1543	5.5636	4.6490
t^5	6.1550	5.1683	5.5802	4.6631
t^6	6.1593	5.1717	5.5834	4.6657
t^7	6.1603	5.1725	5.5844	4.6665
t^8	6.1605	5.1727	5.5846	4.6667
t^9	6.1606	5.1728	5.5847	4.6667
t^{10}	6.1606	5.1728	5.5847	4.6667

Table IV: Global trust in each iteration, when initial guess is very far from global trust ($\alpha = 1/3$)

i	1	2	3	4
t^0	100.0000	300.0000	200.0000	100.0000
t^1	16.9093	12.1379	13.7913	11.3982
t^2	7.6469	6.5685	7.1369	5.9630
t^3	6.5419	5.4824	5.9029	4.9291
t^4	6.2499	5.2477	5.6685	4.7377
t^5	6.1829	5.1918	5.6048	4.6834
t^6	6.1662	5.1775	5.5897	4.6710
t^7	6.1620	5.1740	5.5859	4.6678
t^8	6.1610	5.1731	5.5850	4.6670
t^9	6.1607	5.1729	5.5847	4.6668
t^{10}	6.1606	5.1728	5.5847	4.6667
t^{11}	6.1606	5.1728	5.5847	4.6667

- [2] E. Seneta, *Non-negative Matrices and Markov Chains*, 2nd ed. Springer-Verlag, New York Heidelberg Berlin, 1981.
- [3] Denis Serre, *Matrices Theory and Applications*, Springer-Verlag New York, Inc., 2002.
- [4] Robert B. Ash, *Probability and Measure Theory*, 2nd ed. Chapter 2 pp. 84-85, Academic Press, 2000.
- [5] S. D. Kamvar, M. T. Schlosser and H. Garcia-Molina, "The eigentrust algorithm for reputation management in P2P networks," *Proc. of the 12th international conference on World Wide Web*, ser. WWW '03. New York, USA: ACM, pp. 640-651, 2003, .
- [6] L. Xiong and L. Liu, "Peertrust: Supporting Reputation-Based Trust for Peer-to-Peer Ecommerce Communities," *IEEE Trans. Knowledge and Data Eng.*, vol. 16, no. 7, pp. 843-857, July 2004.
- [7] S. Song, K. Hwang, R. Zhou and Y. K. Kwok, "Trusted P2P Transactions with Fuzzy Reputation Aggregation," *IEEE Internet Computing*, vol. 9, no. 6, pp. 24-34, November-December 2005.
- [8] R. Zhou and K. Hwang, "Powertrust: A Robust and Scalable Reputation System for Trusted Peer-to-Peer Computing," *IEEE Trans. Parallel and Distributed Systems*, vol. 18, no. 4, pp. 460-473, April 2007.

Table V: Global trust in each iteration for lesser value of α ($\alpha = 1/6$)

i	1	2	3	4
t^0	1.0000	2.0000	3.0000	4.0000
t^1	5.4761	4.6006	4.5168	3.4374
t^2	6.1901	5.0137	5.4742	4.5092
t^3	6.2506	5.1176	5.5682	4.5424
t^4	6.2693	5.1288	5.5767	4.5486
t^5	6.2714	5.1304	5.5790	4.5510
t^6	6.2716	5.1307	5.5793	4.5511
t^7	6.2717	5.1307	5.5793	4.5512
t^8	6.2717	5.1307	5.5793	4.5512